Math 1050 – 3.5 Notes

## Graphing Techniques: Transformations

**Parent Graph:** $y = f(x)$

**Offspring:** Transformations of the parent graph.

| | $f(x)=x^2$ | $f(x)=\sqrt{x}$ | $f(x)=\frac{1}{x}$ | Effect on Parent Graph |
|---|---|---|---|---|
| $y=f(x)+2$ | $y=x^2+2$ | $y=\sqrt{x}+2$ | $y=\frac{1}{x}+2$ | ↑ 2 |
| $y=f(x)-2$ | $y=x^2-2$ | $y=\sqrt{x}-2$ | $y=\frac{1}{x}-2$ | ↓ 2 |
| $y=f(x+2)$ | $y=(x+2)^2$ | $y=\sqrt{x+2}$ | $y=\frac{1}{x+2}$ | ← 2 |
| $y=f(x-2)$ | $y=(x-2)^2$ | $y=\sqrt{x-2}$ | $y=\frac{1}{x-2}$ | → 2 |
| $y=2f(x)$ | $y=2x^2$ | $y=2\sqrt{x}$ | $y=\frac{2}{x}$ | vertical stretch (multiply y-coords. by 2) |
| $y=\frac{1}{2}f(x)$ | $y=\frac{1}{2}x^2$ | $y=\frac{\sqrt{x}}{2}$ | $y=\frac{1}{2x}$ | vertical compression (multiply y-coords. by ½) |
| $y=f(2x)$ | $y=(2x)^2$ | $y=\sqrt{2x}$ | $y=\frac{1}{2x}$ | horizontal compression (divide x-coords. by 2) |
| $y=f(\frac{1}{2}x)$ | $y=(\frac{1}{2}x)^2$ | $y=\sqrt{\frac{x}{2}}$ | $y=\frac{1}{\frac{1}{2}x}=\frac{2}{x}$ | horizontal stretch (divide x-coords. by ½) |
| $y=-f(x)$ | $y=-x^2$ | $y=-\sqrt{x}$ | $y=-\frac{1}{x}$ | reflect over x-axis |
| $y=f(-x)$ | $y=(-x)^2$ | $y=\sqrt{-x}$ | $y=\frac{1}{-x}$ | reflect over y-axis |

**When graphing a transformed graph based on an equation, apply transformations in the following order:**

1. reflections
2. stretches/compressions
3. translations (shifts)

You MUST factor out any coefficient of x to figure out horizontal shifts.

**Examples:** List the transformations in the appropriate order:

Parent graph: $y=\sqrt{x}$

a) $y=-\frac{1}{2}\sqrt{x+3}$

1. reflect over x-axis
2. vertical compression by factor of ½
3. shift left 3

b) $y=5\sqrt{-4x}+3$

1. reflect over y-axis
2. vertical stretch by a factor of 5
3. horizontal compression by a factor of ¼
4. shift up 3

c) $y=3\sqrt{-2x+9}$ → $3\sqrt{-2(x-9/2)}$

1. Reflect over y-axis
2. Vertical stretch by a factor of 3
3. Horizontal compression by a factor of ½
4. Shift right 9/2

Parent graph: $f(x)=|x|$

a) $f(x)=4|x-2|+7$

1. Vertical stretch by a factor of 4
2. Shift right 2 & up 7

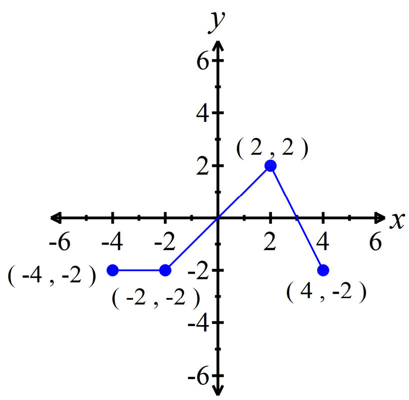
b) $f(x)=-|x+5|-3$

1. Reflect over x-axis
2. Shift left 5 & down 3

c) $f(x)=-|\frac{x}{3}+2|$ → $-|\frac{1}{3}(x+6)|$

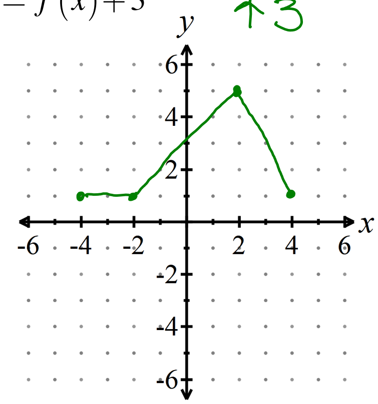
1. Reflect over x-axis
2. Horizontal stretch by a factor of 3
3. Shift left 6

**Example:** The graph of a function $f$ is illustrated below. Use the graph of $f$ as the first step towards graphing each of the following functions:



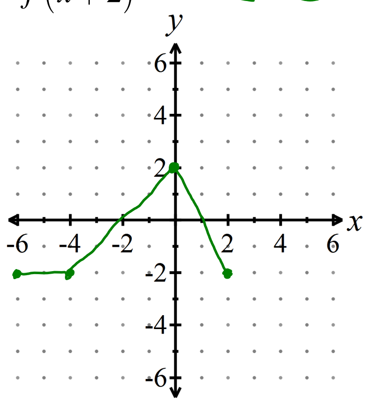

a) $F(x) = f(x) + 3$

↑3



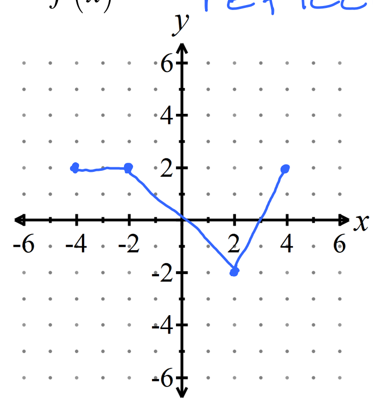
b) $G(x) = f(x+2)$

← 2



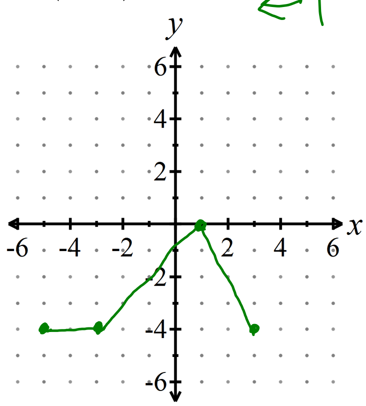
c) $P(x) = -f(x)$

reflect over x-axis



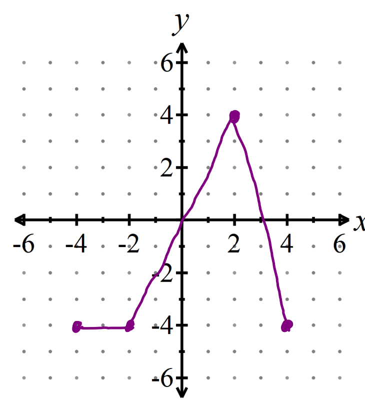
d) $H(x) = f(x+1) - 2$

←1 ↓2



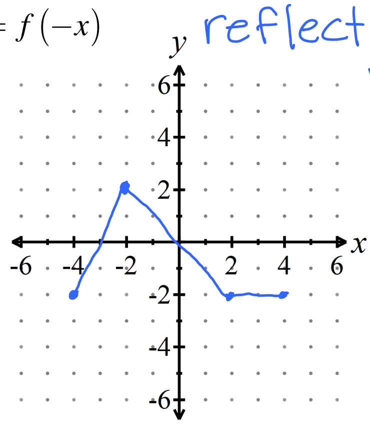
e) $Q(x) = 2f(x)$

vertical stretch: Multiply y-coords. by 2



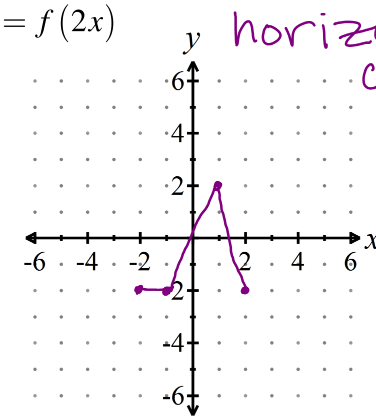
f) $g(x) = f(-x)$

reflect over y-axis



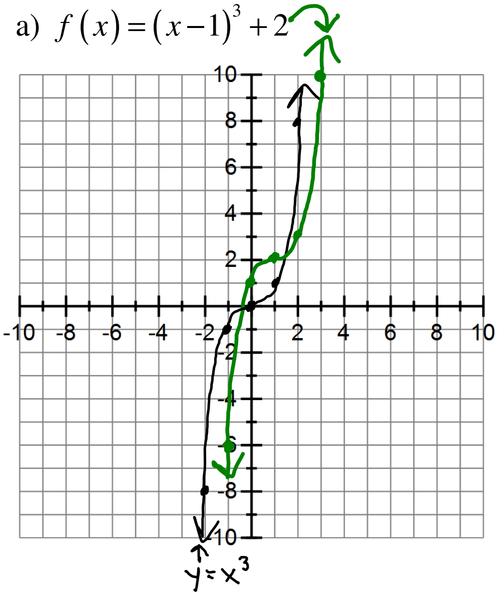
g) $h(x) = f(2x)$

horizontal compression: Divide x-coords. by 2



**Examples:** Graph the following:

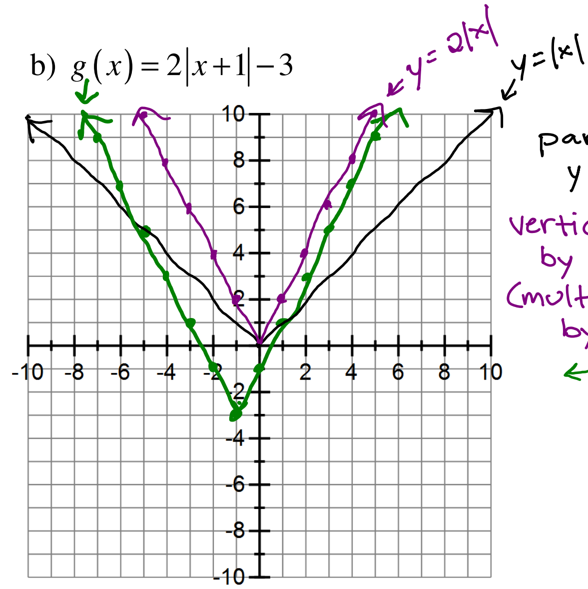
a) $f(x) = (x-1)^3 + 2$




parent: $y = x^3$

$\rightarrow 1 \quad \uparrow 2$

b) $g(x) = 2|x+1| - 3$

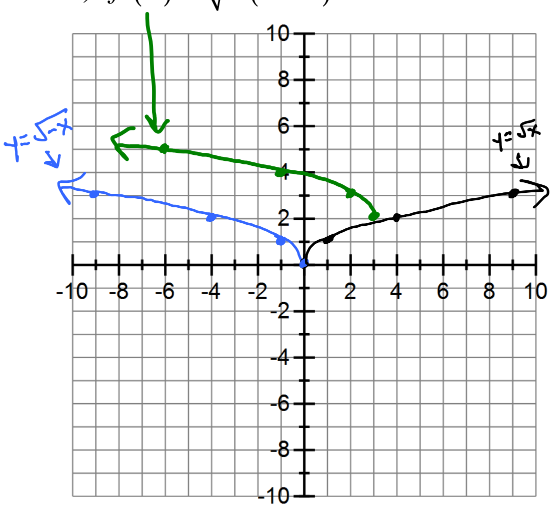



parent: $y = |x|$

vertical stretch by factor of 2 (multiply y-coords. by 2)

$\leftarrow 1 \quad \downarrow 3$

c) $f(x) = \sqrt{-(x-3)} + 2$

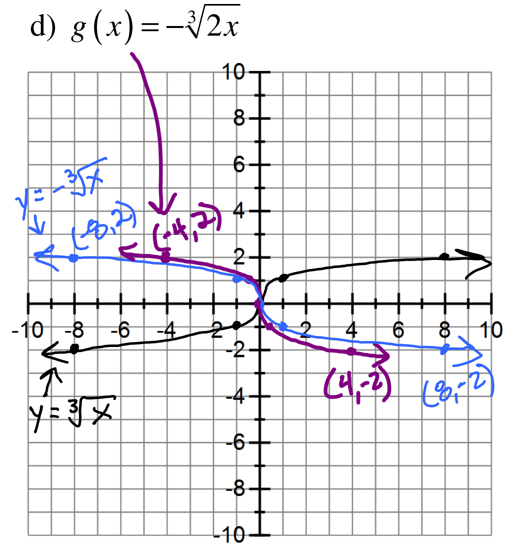



parent: $y = \sqrt{x}$

reflect over y-axis

$\rightarrow 3 \quad \uparrow 2$

d) $g(x) = -\sqrt[3]{2x}$

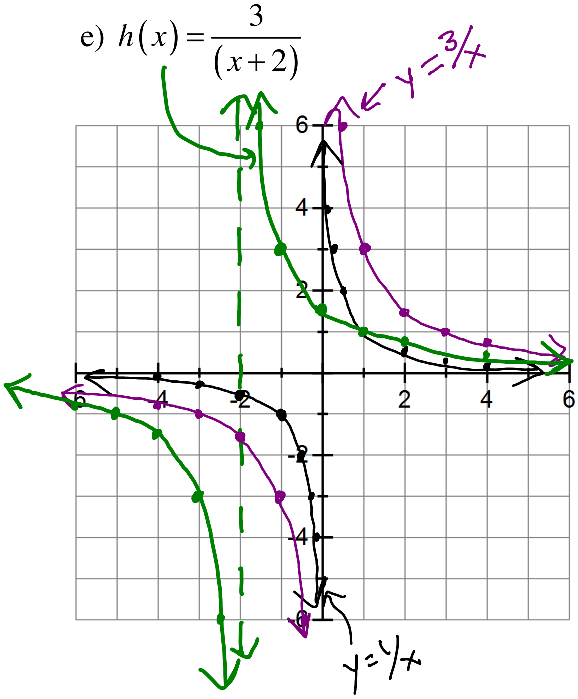



parent: $y = \sqrt[3]{x}$

reflect over x-axis

horizontal compression (divide x-coords. by 2)

e) $h(x) = \dfrac{3}{(x+2)}$




parent: $y = \dfrac{1}{x}$

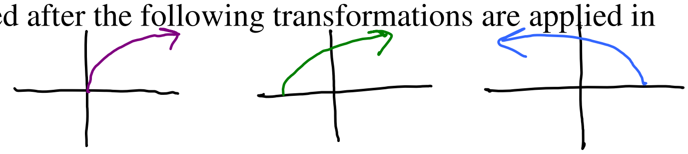
vertical stretch by factor of 3 (multiply y-coords. by 3)

$\leftarrow 2$

**Example:** Write the equation of the function that is graphed after the following transformations are applied in order to the graph of $g(x) = x^3$.

1. Shift down 4 units $y = x^3 - 4$
2. Reflect across $y$-axis $y = (-x)^3 - 4$
3. Vertical compression by a factor of 1/2 $y = \frac{1}{2}(-x)^3 - 4$ or $y = -\frac{1}{2}x^3 - 4$

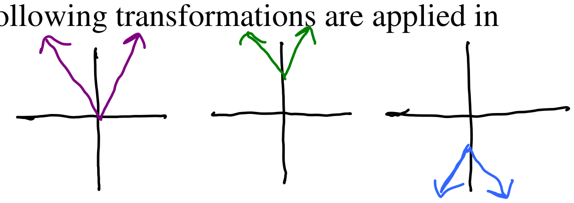
**Example:** Write the equation of the function that is graphed after the following transformations are applied in order to the graph of $h(x) = \sqrt{x}$.



1. Vertical stretch by a factor of 3 $y = 3\sqrt{x}$
2. Move left 5 units $y = 3\sqrt{x+5}$
3. Reflect across the $y$-axis $y = 3\sqrt{-x+5}$ or $y = 3\sqrt{-(x-5)}$

Moving left then reflecting over the y-axis. has the same effect as reflecting over the y-axis, then moving right.

**Example:** Write the equation of the function that is graphed after the following transformations are applied in order to the graph of $f(x) = |x|$.



1. Horizontal compression by a factor of 1/2 $y = |2x|$
2. Move up 6 units $y = |2x| + 6$
3. Reflect across the $x$-axis $y = -(|2x| + 6)$ or $y = -|2x| - 6$

Moving up then reflecting over the x-axis. has the same effect as reflecting over the x-axis, then moving down.

**Summary of Graphing Transformations:**

| To Graph: | Draw the Graph of $y = f(x)$ and: | Functional Change to $y = f(x)$: |
|---|---|---|
| **Reflection About the *x*-axis** $y = -f(x)$ | Reflect the graph of $f$ about the $x$-axis. | Multiply $f(x)$ by $-1$. |
| **Reflection About the *y*-axis** $y = f(-x)$ | Reflect the graph of $f$ about the $y$-axis. | Replace $x$ by $-x$. |
| **Vertical Stretches & Compressions** $y = af(x),\ a > 0$ | Multiply each $y$-coordinate of $y = f(x)$ by $a$. This stretches the graph of $f$ vertically if $a > 1$. This compresses the graph of $f$ vertically if $0 < a < 1$. | Multiply $f(x)$ by $a$. |
| **Horizontal Stretches & Compressions** $y = f(bx), b > 0$ | Divide each $x$-coordinate of $y = f(x)$ by $b$. This stretches the graph of $f$ horizontally if $0 < b < 1$. This compresses the graph of $f$ horizontally if $b > 1$. | Replace $x$ by $bx$. |
| **Vertical Shifts** $y = f(x) + k,\ k > 0$ | Raise the graph of $f$ by $k$ units. | Add $k$ to $f(x)$. |
| $y = f(x) - k,\ k > 0$ | Lower the graph of $f$ by $k$ units. | Subtract $k$ from $f(x)$. |
| **Horizontal Shifts** $y = f(x-h),\ h > 0$ | Shift the graph of $f$ to the right by $h$ units. | Replace $x$ by $x - h$. |
| $y = f(x+h),\ h > 0$ | Shift the graph of $f$ to the left by $h$ units. | Replace $x$ by $x + h$. |